MATHEMATICS

PART - A

Answer all the ten questions : 10 \times 1 = 10

1. Find an integer \( x \), satisfying \( 5x \equiv 4 \pmod{13} \).

2. If the matrix \[
\begin{bmatrix}
6 & x - 2 \\
3 & x
\end{bmatrix}
\] is singular, find \( x \).

3. On the set \( Z \) of integers if "\( O \)" is defined by \( a \circ b = a + b + 1 \), \( \forall a, b \in Z \), find the identity element.

4. If \( \vec{a} = 2\hat{i} + 3\hat{j} \) and \( \vec{b} = 3\hat{i} + 4\hat{j} \), find the magnitude of \( \vec{a} + \vec{b} \).

5. Write the condition (in terms of \( g, f \) and \( c \)) under which \( x^2 + y^2 + 2gx + 2fy + c = 0 \) becomes a point circle.

6. Find the equation of the directrix of the parabola \( y^2 = 8x \).

7. Find the value of \( \sin \left[ \frac{\pi}{2} - \sin^{-1} \left( \frac{-\sqrt{3}}{2} \right) \right] \).

8. Find the modulus of the complex number \( \frac{2 - i}{5i} \).

9. If \( f(x) = x^2 + \frac{1}{x^2} \), find \( f'(1) \).

10. Evaluate \( \int_{0}^{\pi/4} \sin^3 x \cos x \, dx \).
PART - B

Answer any ten questions: \(10 \times 2 = 20\)

11. If \( a \equiv b \pmod{m} \) and \( n > 1 \) is a positive divisor of \( m \), prove that
\[ a \equiv b \pmod{n}. \]

12. Evaluate
\[
\begin{vmatrix}
-a^2 & ab & ac \\
ab & -b^2 & bc \\
ac & bc & -c^2
\end{vmatrix}
\]

13. Define the binary operation, on a non-empty set \( S \). Give an example to show that, on \( \mathbb{Z} \), the operation \(*\), defined by \( a * b = a^b \), is not binary.

14. Find the angle between the vectors \( 2\hat{i} - 2\hat{j} + \hat{k} \) and \( 2\hat{i} - \hat{j} - 2\hat{k} \).

15. Examine whether the point \((1, 5)\) lies outside, inside or on the circle
\[ x^2 + y^2 + 4x + 2y + 3 = 0. \]

16. The two ends of the major axis of an ellipse are \((5, 0)\) and \((-5, 0)\). If \(3x - 5y - 9 = 0\) is a focal chord, find the eccentricity of the ellipse.

17. Prove that \(2 \tan^{-1} \frac{1}{2} + \sin^{-1} \frac{3}{5} = \frac{\pi}{2}\).
18. If \( x = \cos \alpha + i \sin \alpha \) and

\[ y = \cos \beta + i \sin \beta \]

prove that \( \frac{y^3}{x^2} + \frac{x^2}{y^3} = 2 \cos (3\beta - 2\alpha) \).

19. If \( x^y = a^x \), prove that \( \frac{dy}{dx} = \frac{x \log a - y}{x \log x} \).

20. Find the length of the subtangent to the curve \( y = \sqrt{x^2 + x + 1} \) at the point \( (1, \sqrt{3}) \) on it.

21. Integrate \( \sin 3x \cos x \) with respect to \( x \).

22. Form the differential equation of the family of straight lines passing through the origin of Cartesian plane.

**PART - C**

I. Answer any three questions: \( 3 \times 5 = 15 \)

23. Find the G.C.D. of 408 and 1032 using Euclidean algorithm. Express it in two ways in the form \( 408m + 1032n \) where \( m, n \) are integers. \( 5 \)
24. a) Find $x$ and $y$ if
\[
\begin{bmatrix}
  x & 2 & -3 \\
  5 & y & 2 \\
  1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
  3 & -1 & 2 \\
  4 & 2 & 5 \\
  2 & 0 & 3
\end{bmatrix}
= \begin{bmatrix} 5 & 3 & 3 \\ 19 & -5 & 10 \\ 1 & -3 & 0 \end{bmatrix}
\]

b) Solve by Cramer's rule:

\[2x - y = 10\]

\[x - 2y = 2.\]

25. a) Given that $H$ is a non-empty subset of a set $G$ and $(G, \ast)$ is a group. If for all $a, b \in H$, $a \ast b^{-1} \in H$, prove that $(H, \ast)$ is a subgroup of $(G, \ast)$.

b) If, in a group $G$, every element is its own inverse, prove that $G$ is an Abelian group.

26. a) Using vector method, find the area of the triangle whose vertices are $(1, 2, 3)$, $(2, -1, 1)$ and $(1, 2, -4)$.

b) Find the volume of the parallelopiped whose co-terminal edges are $2\hat{i} + \hat{j} - \hat{k}$, $3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\hat{i} - 3\hat{j} - 3\hat{k}$.

II. Answer any two questions:

27. a) Find the equation of the circle which passes through the point $(2, 3)$, has its centre on $x + y = 4$ and cuts orthogonally the circle $x^2 + y^2 - 4x + 2y - 3 = 0$. 

www.pucbiology.wordpress.com
b) Find the radical centre of the circles $x^2 + y^2 + 2x - 4 = 0$, $x^2 + y^2 + 4y - 4 = 0$ and $x^2 + y^2 - 2x - 5 = 0$. 2

28. a) Find the centre and the eccentricity of the hyperbola

$$x^2 - 3y^2 - 4x + 6y + 11 = 0.$$ 3

b) Find the equation of the parabola with vertex (-4, 2), axis $y = 2$ and passing through the point (0, 6). 2

29. a) If $x \geq 0$ and $y \geq 0$, prove that

$$\sin^{-1} x - \sin^{-1} y = \sin^{-1} \left( x \sqrt{1 - y^2} - y \sqrt{1 - x^2} \right).$$ 3

b) Find the general solution of the equation

$$\cos x - \cos 7x = \sin 4x.$$ 2

III. Answer any three of the following questions: 3 x 5 = 15

30. a) Differentiate $e^x$ with respect to $x$ from first principles. 3

b) Differentiate $\log_{10} (\log x)$ with respect to $x$. 2

31. a) If $y = x \cosh x$, prove that

$$xy_2 - 2y_1 - xy + 2 \cosh x = 0.$$ 3
b) Prove that $x^x$ function has a minimum value at $x = \frac{1}{e}$.  

32. a) Find \( \int \frac{x + 1}{x^2 - 4x + 6} \, dx \).  

b) A stone is thrown up vertically and the height \( x \) feet reached by it in time \( "t" \) seconds is given by \( x = 80t - 16t^2 \). Find the time for the stone to reach its maximum height. Also find the maximum height reached by the stone.  

33. a) If \( x = a(\theta + \sin \theta) \) and \( y = a(1 - \cos \theta) \), find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \).  

b) Find \( \int \frac{xe^x}{(1 + x^2)^2} \, dx \).  

34. Find the area bounded by the curves \( 4y^2 = 9x \) and \( 3x^2 = 16y \).  

\[ \text{PART - D} \]

Answer any two of the following questions:  

\[ 2 \times 10 = 20 \]

35. a) Define ellipse as the locus of a point. Derive the equation of the ellipse in the form \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) \( (a > b) \).
b) Using Caley-Hamilton theorem, find the inverse of the matrix
\[
\begin{bmatrix}
1 & 2 \\
3 & 1
\end{bmatrix}
\].

36. a) Find all the cube roots of the complex number \(\sqrt[3]{3} + i\). Represent them in the Argand diagram. Find their product.

b) Prove by vector method that the medians of a triangle are concurrent.

37. a) A man 6 feet tall moves away from a source of light 20 feet above the ground level and his rate of walking being 4 miles/hour. At what rate is the length of the shadow changing? At what rate is the tip of the shadow moving?

b) Find the general solution of
\[
\sqrt{3} \cos x + \sin x - \sqrt{2}.
\]

38. a) Evaluate \[\int_0^{\pi/2} \log \sin x \, dx\]

b) Find the general solution of the differential equation
\[y \log x \cdot \log y \, dx + dy = 0.\]